Uisrael
REVISTA CIENTÍFICA

May – August 2025 period
Vol. 12, No. 2
rcientifica@uisrael.edu.ec
https://doi.org/10.35290/rcui.v12n2.2025.1297 • e-ISSN: 2631 - 2786

Learning of areas with double integrals using GeoGebra: A study of master's students in mathematics education

Aprendizaje de áreas con integrales dobles usando GeoGebra: Un estudio en alumnos de maestría en educación matemática.

Date of receipt: 2024-05-27 Date of acceptance: 2024-11-20 Date of publication: 2025-05-10

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Abstract

The main purpose of this research was to evaluate the impact of the teaching of area calculation using Double Integrals using GeoGebra in students enrolled during Geometric and Trigonometric Analysis within the master's degree in education, mention in Mathematics Teaching, at the Universidad Técnica Particular de Loja. From this perspective, the research approach was framed in the positivist paradigm and adopted a quantitative approach of descriptive correlational character. The sample was selected from a population of 70 students, who were distributed in two groups, 35 students for the experimental group and 35 for the control group. A study sequence was implemented with the purpose of applying a dynamic geometric representation of the concept of Double Integral through the parameterization of curves and surfaces in GeoGebra, which allowed geometric visualization and calculation of generated areas. As a result, a significant improvement was observed in the geometric understanding of the Double Integral by the students with respect to the calculation of areas in flat regions..

Keywords: areas, double integrals, GeoGebra, academic performance

Resumen

El propósito fundamental de esta investigación fue evaluar el impacto de la enseñanza del cálculo de áreas a través del uso de Integrales Dobles mediante GeoGebra en los estudiantes inscritos en el curso de Análisis Geométrico y Trigonométrico dentro de la Maestría en Educación mención Enseñanza de la Matemática de la Universidad Técnica Particular de Loja. Desde esta perspectiva, el enfoque de la investigación se enmarcó en el paradigma positivista y adoptó un enfoque cuantitativo de carácter descriptivo correlacional. La muestra se seleccionó de 70 estudiantes, distribuidos en dos grupos, 35 estudiantes para el grupo experimental y 35 para el de control. Se implementó una secuencia de estudio para aplicar una representación geométrica dinámica del concepto de Integral Doble mediante la parametrización de curvas y superficies en GeoGebra, lo que permitió visualizar geométrica y calcular áreas generadas. Como resultado, se observó una mejora significativa en la comprensión geométrica de la Integral Doble por parte de los estudiantes con respecto al cálculo de áreas en regiones planas.

Palabras clave: áreas, integrales dobles, GeoGebra, rendimiento académico

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Introduction

The integral calculus of functions of one or more variables is usually considered by students at both undergraduate and graduate level as one of the most complex topics within mathematics courses (Pino et al., 2018), due to the fact that it is not enough to follow algebraic procedures to correctly calculate the result of an integral, but it is essential to understand the geometric meaning of the mathematical object.

Frequently, university students in the first semesters taking subjects related to mathematics respond to abstract contents in a memoristic way by solving the exercises mechanically, forgetting how important it is to delve into the concepts that enclose the topics in question.

Given the abstract nature of calculus, the use of computational tools, such as GeoGebra, is essential in the teaching of double integrals. In traditional approaches, mathematical graphs are represented statically, usually plotted on the blackboard or on paper, which limits dynamic interaction with geometric representations of the regions of integration. This restriction prevents a more accurate and manipulable visualization of mathematical objects, negatively affecting the deep understanding of concepts such as calculating areas and volumes using multiple integrals (Dahl et al., 2019).

The disconnection between geometry and algebra in the study of mathematical objects generates that students with less ability in abstract thinking get lost in an environment dominated by formulas and equations, which are applied mechanically without considering their geometric interpretation (Baena, 2020). As a result, students tend to memorize common exercises, facing difficulties when presented with problems that require more logical and spatial understanding. This situation is due to the fact that they do not go deep enough into the geometric representation of the mathematical object, which limits their ability to continue with a rigorous and complete analysis (Quintilla and Fernández, 2021).

According to Duval (2006), it is always necessary to move from one semiotic register to another to the mathematical object studied, because if the student only remains in one register, for example, the algebraic one, he will not be able to interpret it geometrically. In the same way, Svensson and Campos (2022) state that it is essential for the student to be able to move from one register to another, which will allow him to recognize with greater agility each register that represents the mathematical object.

The use of technological tools, such as GeoGebra, is essential for the conversion between different registers of semiotic representation. This software allows the study of various mathematical objects and facilitates the user's transition between the algebraic and graphical registers, and vice versa. GeoGebra, developed in 2002 by Markus Hohenwarter as part of his master's thesis at the University of Salzburg, Austria (Arteaga et al., 2019), has been consolidated since its inception as an open source software, standing out for its accessibility and ease of use.

According to Ortiz (2019), the use of GeoGebra in university students has a significant impact on their ability to understand exercises related to the representation of regions and the calculation of volumes. Similarly, León (2021) points out that the incorporation of augmented reality with GeoGebra in the learning process of spatial geometry generates a positive effect on elementary school students. Finally, Narh and Sabtiwu (2022) highlight that the use of GeoGebra in the teaching and learning of geometry produces a noticeable improvement in the grades and interest of both mathematics education students and teachers, based on an applied research approach.

Therefore, the use of technological tools requires teachers to ensure that their classes are participatory, dynamic and engaging. This is relevant when teaching a digital native generation, with a natural mastery of technology, having been born and raised on digital devices and technological resources. The effective integration of these tools in the classroom not only fosters active learning, but also responds to the expectations and needs of students accustomed to interacting with technology from an early age (Jiménez and Jiménez, 2017).

In the master's degree in Mathematics Education and in other careers at the Universidad Técnica Particular de Loja, new teaching and learning strategies are sought that are conducive to obtaining better results and allow increasing the critical thinking indispensable in learners, which is a permanent challenge for the teacher seeking methodologies that contemplate the use of ICT in their classes.

In the subject of Geometric and Trigonometric Analysis, the topics are very relevant and complex due to their extensive mathematical contents. This is evident in the double integral when it comes to represent algebraically and geometrically certain concepts, especially when the study of flat regions is approached. For this reason, this work proposes the use of GeoGebra software in the didactics and understanding of the double integral, in the geometric analysis of the calculation of areas of flat regions of Type I and Type II.

1.1. Fundamental Mathematical Concepts of Double Integrals

Antes de introducir la herramienta tecnológica de GeoGebra en la enseñanza de las integrales dobles, es esencial establecer una base sólida en los conceptos matemáticos fundamentales que subyacen a este tema. A continuación, se presentan los conceptos clave que los estudiantes deben comprender para abordar las integrales dobles de manera efectiva.

1.1.1. Definition of Double Integral

A double integral is used to calculate the volume under a surface in three-dimensional space. It is defined as the extension of the simple integral to functions of two variables. Mathematically, the double integral of a function $f(x,y)$ over a region R in the plane is expressed as:

$$\iint_R f(x,y)dA$$

where dA represents an area element in the region R. This integral can be interpreted as the sum of infinite contributions of $f(x,y)$ at each point in the region *R*.

However, this study will focus only on the calculation of areas of flat regions, i.e., double integrals without function $f(x,y)$, which are calculated as follows:

$$\iint_R dA$$

1.1.2. Integration Regions

Double integrals can be computed over different types of regions in the plane, which are generally classified into two types:

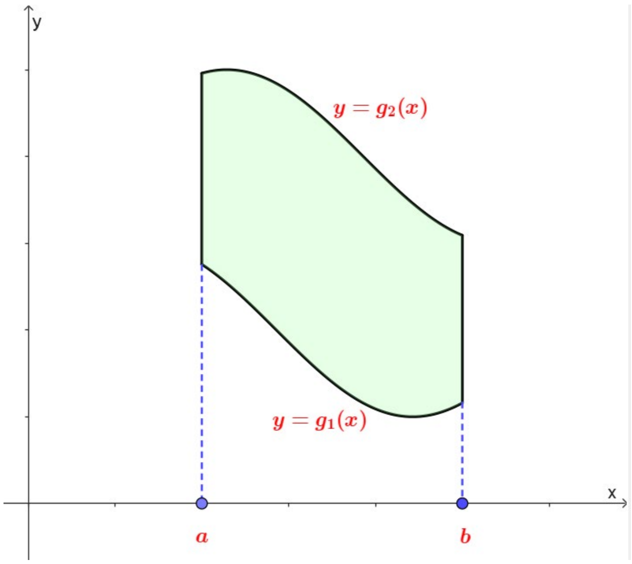
- **Type I regions:** these are regions that can be described as the area between two curves in the *xy*. plane. They are integrated first with respect to y and then with respect to *x*.

$$\int_a^b \int_{y=g_1(x)}^{y=g_2(x)} dydx$$

This type of region is illustrated in figure 1

Figure 1

Type I Region



Note: Own elaboration

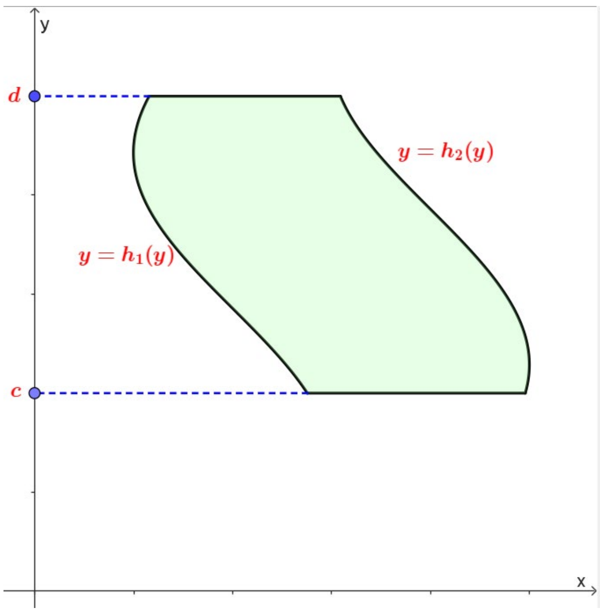
- **Type II Regions:** These are areas that can be described as the area between two horizontal lines. They are integrated first with respect to *x* and then with respect to *y*.

$$\int_{c}^{d} \int_{x=h_1(y)}^{y=h_2(y)} dxdy$$

These type II regions are illustrated in Figure 2.

Figure 2

Type II Region



Note: Own elaboration

The correct identification of the region of integration is crucial to establish the limits of integration in the calculation of areas through double integrals.

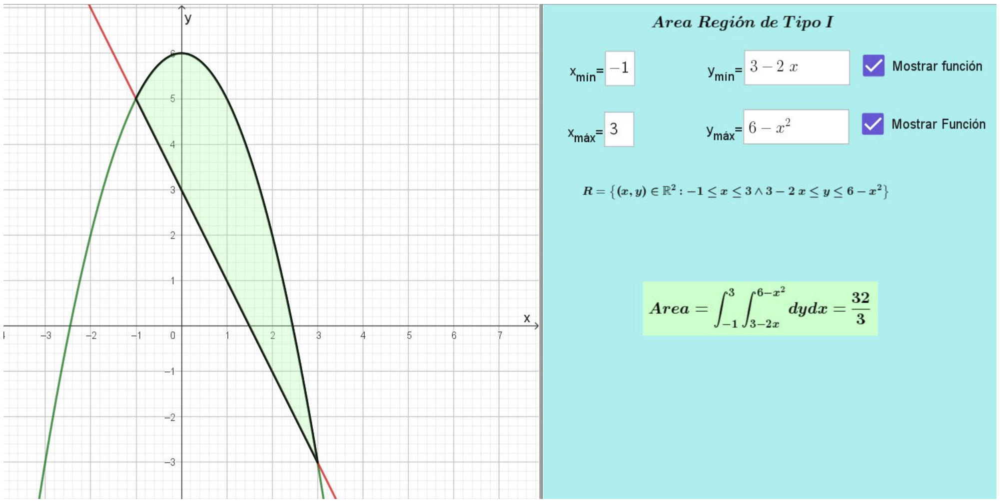
1.2.3. Geometric Interpretation

It is essential that students understand the geometric interpretation of double integrals. This includes visualizing how a no-argument integral f(x,y) represents the area of the region R. Understanding this relationship between algebra and geometry is key to effective learning of double integrals. Therefore, specific applets have been developed to calculate areas in type I and type II regions, so that students can dynamically visualize the regions and obtain the value of them. These applets are shown in Figures 3 and 4, and can be accessed through the following link:

https://utpl-my.sharepoint.com/:f:/g/personal/rlluna_utpl_edu_ec/Ej2jHFTanw5Aqi1J2Jz0LnoBd5rb1PZmjwDlmZukhp3-2Q?e=KS1iLL

Figure 3

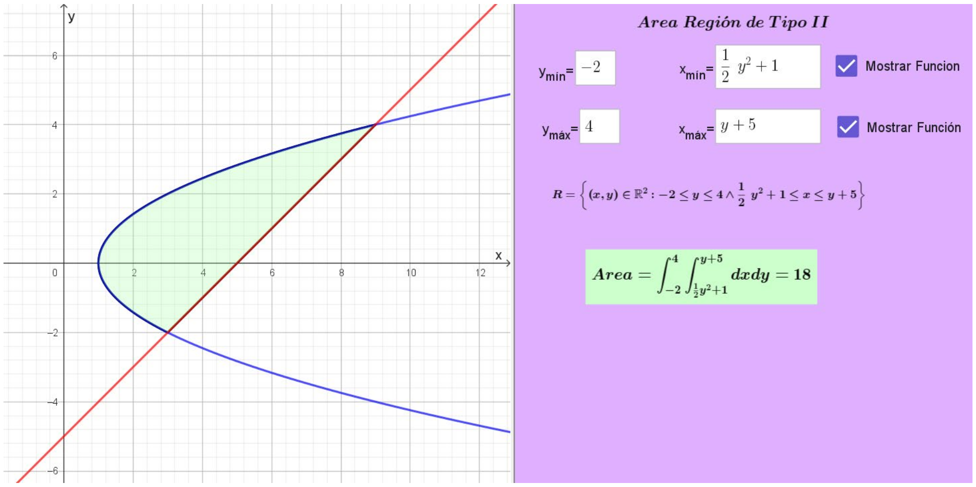
Area by double integral of a type I region



Note: Own elaboration

Figure 4

Area by double integral of a type II region



Note: Own elaboration

Methodology

In this research work, the positivist paradigm was chosen, since it provides a clear separation between the researcher, understood as a neutral subject, and the study work, independent of the researcher's subjectivity (Miranda and Ortiz, 2020).

The research was quantitative, since variables were used to measure the results numerically.

The scope of the research was descriptive-correlational. It was descriptive because it focused on specifying the characteristics of the population under study (Guevara et al., 2020), while it was correlational because it sought to measure or collect information regarding the study variables (GeoGebra and student learning) to then determine the incidence or relationship that exists of one variable with respect to the other within the sample (Hernández et al., 2014).

A quasi-experimental research design was chosen because of its focus on analyzing the causality between the independent variable (GeoGebra use) and the dependent variable (student learning) (Valmi et al., 2007).

Another reason for selecting a quasi-experimental design was that the assignment of students to the study groups was not randomized, since the two courses used as experimental and control group had been previously determined by the Universidad Técnica Particular de Loja (Zurita et al., 2018).

2.1. Objective

To determine the impact generated by the teaching of areas with Double Integrals using GeoGebra software in students taking the Geometric and Trigonometric Analysis course of the master's degree in education, mention in mathematics teaching, at the Universidad Técnica Particular de Loja (UTPL).

2.2. Population and Sample

In the Master's Degree in Education in Mathematics Teaching of the UTPL, the total population in Geometric and Trigonometric Analysis is 70 students distributed in two parallels of 35 students each. Thus, one parallel was chosen as the experimental group, to which the classes with GeoGebra were applied, and the other was considered the control group, which allowed comparing if there is an impact on the teaching of the mathematical object through the GeoGebra software.

2.3. Instrument

The technique and instrument used in this research, after having taught the mathematical object in several class sessions, consisted of the application of a survey by means of a questionnaire, addressed to both the experimental group and the control group. This made it possible to obtain the scores of each student for subsequent statistical analysis.

2.4. Procedure for data collection and analysis

Due to the nature of this research and its design, statistical tools had to be used to process and analyze the data and interpret the results in order to respond to the stated objective.

After data collection using the instrument of this research, the values were organized in Excel tables and then transferred to SPSS (Statistical Package for the Social Sciences) software.

According to Purwanto et al. (2021), SPSS is one of the most widely used programs in quantitative research, as it facilitates researchers to accurately organize and analyze large volumes of data.

Results

Several sessions of area classes with double integrals were implemented by means of a traditional class to the control group and by means of a dynamic geometric representation through applets, elaborated with GeoGebra, to the experimental group.

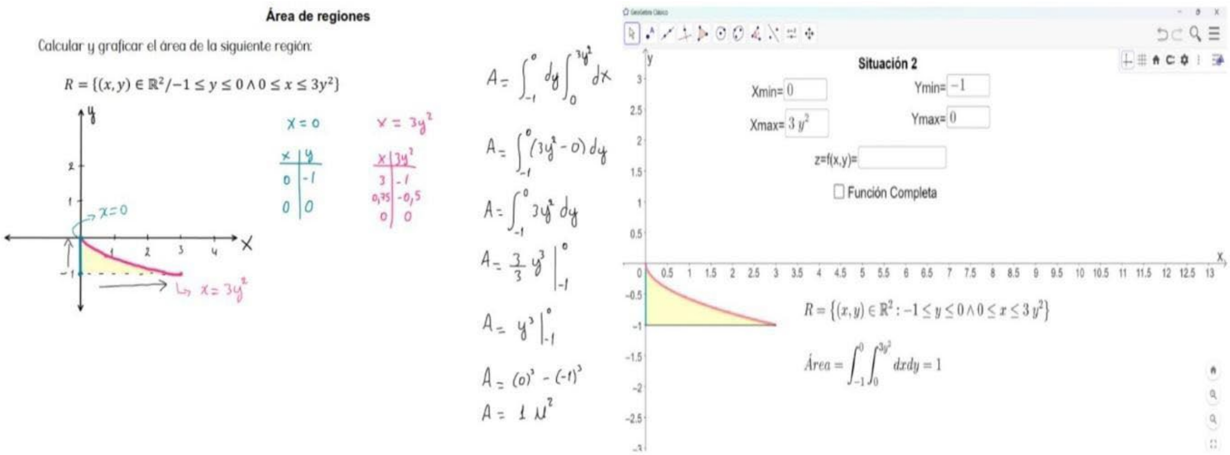
In the applets shown in Figures 3 and 4, students enter the values of the functions and the interval that make up the region, and as a response, the graph of the region and the value of its area are shown. These applets converted the algebraic register to the graph, thus putting Duval's theory of semiotic representations registers into practice.

This allowed the students in the experimental group to check whether their hand-drawn graphs were correct, as well as to verify the result of the double integral.

Figure 3 shows the workshop of a student in the last class where he performs the calculation of an area with a double integral by hand and its verification with GeoGebra.

Figure 5

Workshop elaborated by a student of the experimental group



Note: Own elaboration

The elaboration of the workshops of the last class by the students of the experimental group of the master's degree in mathematics education is shown in the following link:

https://utpl-my.sharepoint.com/:f:/r/personal/rlluna_utpl_edu_ec/Documents/Taller%20Area%20de%20Regiones?csf=1&web=1&e=jD0L0E

Once the classes were implemented, the students were evaluated with a 10-point questionnaire, according to the criteria in Table 1. These results are presented in Figure 6:

Table 1

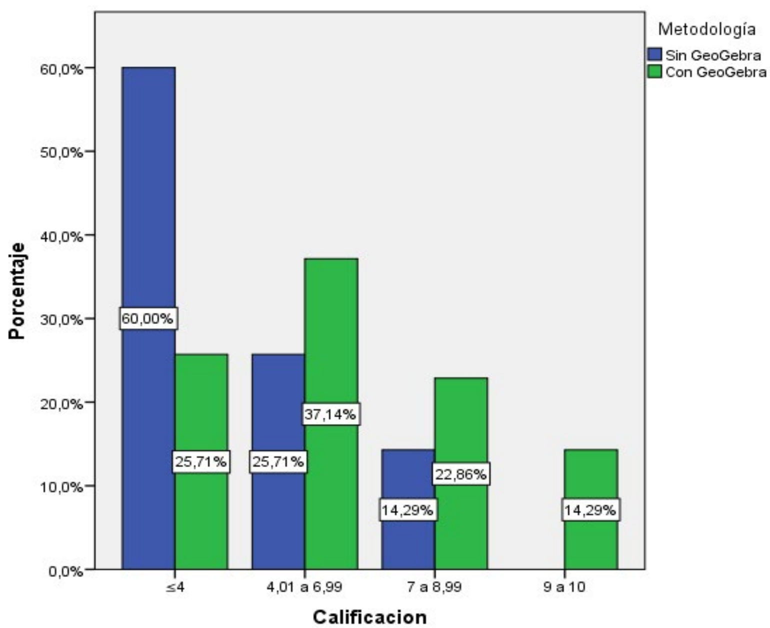
Student evaluation criteria

Llevaron a cabo de manera adecuada la coordinación de registros, prepararon correctamente los límites de integración de la región en el plano y resolvieron la integral.	Sin cometer errores en los cálculos algebraicos. Con imprecisiones en los cálculos algebraicos.
Coordinaron correctamente los registros, pero únicamente en los límites de integración de la región en el plano.	Sin cometer errores en los cálculos algebraicos Con imprecisiones en los cálculos algebraicos.

Note: Own elaboration

Figure 6

Results of the evaluation of the experimental-control group



Note: Own elaboration

This comparative graph shows that the results obtained in the evaluation in the experimental group are better than in the control group, since there is a lower percentage of students with grades lower than or equal to 4 in the experimental group than in the control group, and there is a higher percentage of grades higher than or equal to 7 in the experimental group with respect to the control group, that is, 22.86% in the experimental group compared to 14.29% of students in the control group who obtained grades between 7 and 8.99, and 14.29% of the experimental group compared

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to 0% of the control group with grades between 9 and 10, giving a total difference of 22.86% of students with grades greater than or equal to 7 in the experimental group than in the control group.

Although the results of the experimental group are better than those of the control group, it is necessary to know if there is an incidence of the use of GeoGebra in the academic performance of the students of the Master's Degree in Education in Mathematics Teaching, so a statistical test was performed to determine if there is a significant difference between the means of the experimental group and the control group.

First, the normality test was applied to both the experimental group and the control group. These results are shown in the following table..

Table 2

Normality test in the results obtained from the sample

	Kolmogorov-Smirnov			Shapiro-Wilk		
	Estadístico	**gl**	**Sig.**	**Estadístico**	**gl**	**Sig.**
Resultados Sin GeoGebra	,150	35	,044	,945	35	,081
Resultados con GeoGebra	,123	35	,199	,946	35	,084

Note: Own elaboration

The Shapiro-Wilk test was considered because the number of students in each group is less than 50 and a significance of 0.05, i.e., 95% confidence, was taken as a basis. The p-values shown in Table 1 corresponding to the results without GeoGebra (control group) and with GeoGebra (experimental group) are 0.081 and 0.084 respectively, which indicates that both exceed the 0.05 value of significance; that is, the grades of both groups possess a normal distribution (Hernández and Mendoza, 2018).

Once the grades passed the normality test we proceeded to perform the parametric statistical Student's t-test for independent samples, which is shown in the following table.

Table 3

T-test for the equality of means in the students' evaluation score

		Prueba de Levene para la igualdad de varianzas		Prueba T para la igualdad de medias		
		F	**Sig.**	**t**	**gl**	**Sig. (bilateral)**
Calificación	Se han asumido varianzas iguales	,285	,595	-3,814	68	,000
	No se han asumido varianzas iguales			-3,814	67,476	,000

Note: Own elaboration

Table 2 shows that Levene's test for equality of variances shows that p = 0.595 >= 0.05, which indicates that the variances of the groups are equal. Therefore, the control and experimental groups are homogeneous.

As in the normality test and Levene's test, a confidence interval of 95% was considered, which is equivalent to a significance of 0.05. Thus, the result of the Student's t-test between these two study groups yielded a bilateral significance of 0.000, being this a value lower than 0.05, which mathematically evidences that the use of GeoGebra for teaching the calculation of areas with double integrals significantly improves the academic performance of the students of the master's degree in education, mention in mathematics teaching.

Discussion and conclusions

The superior results of the experimental group compared to the control group, according to Arteaga et al. (2019), are due to the fact that GeoGebra is a valuable tool in the teaching-learning process of mathematics and related disciplines. GeoGebra facilitates rapid mathematical problem solving during learning, stimulates students' creativity, allowing them to explore and build the essential foundations for a deep understanding of any mathematical concept.

However, there is a low percentage of students who still do not master the mathematical object under study and its respective graphical representation, this according to Borja et al.(2021) is due to additional factors that affect the academic performance of students at university level, the main ones being: economic possibilities, problems with the use of new technologies, poor training at high school and undergraduate level and the number of hours devoted to their study.

In this research work it is shown that the application of GeoGebra to teach the calculation of areas with double integrals gave positive results at a general level; it is determined that the use of this program significantly influences the academic performance of the students participating in this study. This is evidenced, above all, by the fact that in the control group the students' grades were less than or equal to 4 in 60%, while in the experimental group only in 25.71%.

Although the application time of GeoGebra was limited due to the cross-sectional nature of the study, its usefulness was clearly demonstrated. This suggests that by increasing the time and frequency of use of the software, even more significant results are likely to be obtained.

Finally, after the analysis carried out, it is concluded that the teaching of the calculation of areas with Double Integrals using GeoGebra software improves the academic performance and therefore the comprehension of the students, so it is confirmed that this technological tool is a valid alternative to improve the academic performance of the university students of the master's degree in education mention in mathematics teaching.

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